

## Natural Healing and Medical Research: Learn the Secrets of Natural Medicine

Area of mathematics

$ax^2 + bx + c = 0$ , where  $a$  is not zero, in terms of its coefficients  $a$ ,  $b$  and  $c$ . The quadratic formula expresses the solution of the equation, where  $a$  is not zero, in terms of its coefficients and

Algebra (from Arabic  $\text{الجبر}$  (al-jabr) 'reunion of broken parts,[1] bonesetting')[2] roughly speaking is the study of mathematical symbols and the rules for manipulating these symbols in formulas;[3] it is a unifying thread of almost all of mathematics.[4]

Elementary algebra deals with the manipulation of variables (commonly represented by Roman letters) as if they were numbers and is therefore essential in all applications of mathematics. Abstract algebra is the name given, mostly in education, to the study of algebraic structures such as groups, rings, and fields (the term is no more in common use outside educational context). Linear algebra, which deals with linear equations and linear mappings, is used for modern presentations of geometry, and has many practical applications (in weather forecasting, for example). There are many areas of mathematics that belong to algebra, some having "algebra" in their name, such as commutative algebra, and some not, such as Galois theory.

The word algebra is not only used for naming an area of mathematics and some subareas; it is also used for naming some sorts of algebraic structures, such as an algebra over a field, commonly called an algebra. Sometimes, the same phrase is used for a subarea and its main algebraic structures; for example, Boolean algebra and a Boolean algebra. A mathematician specialized in algebra is called an algebraist.

Etymology

algebra comes from the title of a book by [5] The word comes from the title of a book by Muhammad ibn Musa al-Khwarizmi

The word algebra comes from the Arabic:  $\text{الجبر}$ , romanized: al-jabr, lit. 'reunion of broken parts,[1] bonesetting[2]' from the title of the early 9th century book  $\text{ilm al-jabr wa l-muqābala}$  "The Science of Restoring and Balancing" by the Persian mathematician and astronomer al-Khwarizmi. In his work, the term al-jabr referred to the operation of moving a term from one side of an equation to the other,  $\text{al-muqābala}$  "balancing" referred to adding equal terms to both sides. Shortened to just algeber or algebra in Latin, the word eventually entered the English language during the 15th century, from either Spanish, Italian, or Medieval Latin. It originally referred to the surgical procedure of setting broken or dislocated bones. The mathematical meaning was first recorded (in English) in the 16th century.[6]

Different meanings of "algebra"

The word "algebra" has several related meanings in mathematics, as a single word

or with qualifiers.

Algebra as a branch of mathematics

Algebra began with computations similar to those of arithmetic, with letters standing for numbers.[7] This allowed proofs of properties that are true no matter which numbers are involved. For example, in the quadratic equation

$$a x^2 + b x + c = 0 , \{displaystyle ax^{2}+bx+c=0,\}$$

$a$  ,  $b$  ,  $c$   $\{displaystyle a,b,c\}$  can be any numbers whatsoever (except that  $a$   $\{displaystyle a\}$  cannot be  $0$   $\{displaystyle 0\}$  ), and the quadratic formula can be used to quickly and easily find the values of the unknown quantity  $x$   $\{displaystyle x\}$  which satisfy the equation. That is to say, to find all the solutions of the equation.

Historically, and in current teaching, the study of algebra starts with the solving of equations, such as the quadratic equation above. Then more general questions, such as "does an equation have a solution?", "how many solutions does an equation have?", "what can be said about the nature of the solutions?" are considered. These questions led extending algebra to non-numerical objects, such as permutations, vectors, matrices, and polynomials. The structural properties of these non-numerical objects were then formalized into algebraic structures such as groups, rings, and fields.

Before the 16th century, mathematics was divided into only two subfields, arithmetic and geometry. Even though some methods, which had been developed much earlier, may be considered nowadays as algebra, the emergence of algebra and, soon thereafter, of infinitesimal calculus as subfields of mathematics only dates from the 16th or 17th century. From the second half of the 19th century on, many new fields of mathematics appeared, most of which made use of both arithmetic and geometry, and almost all of which used algebra.

Today, algebra has grown considerably and includes many branches of mathematics, as can be seen in the Mathematics Subject Classification[8] where none of the first level areas (two digit entries) are called algebra. Today algebra includes section 08-General algebraic systems, 12-Field theory and polynomials, 13-Commutative algebra, 15-Linear and multilinear algebra; matrix theory, 16-Associative rings and algebras, 17-Nonassociative rings and algebras, 18-Category theory; homological algebra, 19-K-theory and 20-Group theory. Algebra is also used extensively in

11-Number theory and 14-Algebraic geometry.

## History

The use of the word "algebra" for denoting a part of mathematics dates probably from the 16th century.[citation needed] The word is derived from the Arabic word al-jabr that appears in the title of the treatise Al-Kitab al-muhtasar fi hisab al-gabr wa-l-muqabala (The Compendious Book on Calculation by Completion and Balancing), written circa 820 by Al-Kwarizmi.

Al-jabr referred to a method for transforming equations by subtracting like terms from both sides, or passing one term from one side to the other, after changing its sign.

Therefore, algebra referred originally to the manipulation of equations, and, by extension, to the theory of equations. This is still what historians of mathematics generally mean by algebra.[citation needed]

In mathematics, the meaning of algebra has evolved after the introduction by Franois Vite of symbols (variables) for denoting unknown or incompletely specified numbers, and the resulting use of the mathematical notation for equations and formulas. So, algebra became essentially the study of the action of operations on expressions involving variables. This includes but is not limited to the theory of equations.

At the beginning of the 20th century, algebra evolved further by considering operations that act not only on numbers but also on elements of so-called mathematical structures such as groups, fields and vector spaces. This new algebra was called modern algebra by van der Waerden in his eponymous treatise, whose name has been changed to Algebra in later editions.

## Early history

The roots of algebra can be traced to the ancient Babylonians,[9] who developed an advanced arithmetical system with which they were able to do calculations in an algorithmic fashion. The Babylonians developed formulas to calculate solutions for problems typically solved today by using linear equations, quadratic equations, and indeterminate linear equations. By contrast, most Egyptians of this era, as well as Greek and Chinese mathematics in the 1st millennium BC,

usually solved such equations by geometric methods, such as those described in the Rhind Mathematical Papyrus, Euclid's Elements, and The Nine Chapters on the Mathematical Art. The geometric work of the Greeks, typified in the Elements, provided the framework for generalizing formulae beyond the solution of particular problems into more general systems of stating and solving equations, although this would not be realized until mathematics developed in medieval Islam.[10]

By the time of Plato, Greek mathematics had undergone a drastic change. The Greeks created a geometric algebra where terms were represented by sides of geometric objects, usually lines, that had letters associated with them.[7] Diophantus (3rd century AD) was an Alexandrian Greek mathematician and the author of a series of books called Arithmetica. These texts deal with solving algebraic equations,[11] and have led, in number theory, to the modern notion of Diophantine equation.

Earlier traditions discussed above had a direct influence on the Persian mathematician Muḥammad ibn Mūsā al-Khwarizmi ( c. 780–850). He later wrote The Compendious Book on Calculation by Completion and Balancing, which established algebra as a mathematical discipline that is independent of geometry and arithmetic.[12]

The Hellenistic mathematicians Hero of Alexandria and Diophantus[13] as well as Indian mathematicians such as Brahmagupta, continued the traditions of Egypt and Babylon, though Diophantus' Arithmetica and Brahmagupta's Brāhmasphuṭasiddhānta are on a higher level.[14][better source needed] For example, the first complete arithmetic solution written in words instead of symbols,[15] including zero and negative solutions, to quadratic equations was described by Brahmagupta in his book Brahmasphutasiddhanta, published in 628 AD.[16] Later, Persian and Arab mathematicians developed algebraic methods to a much higher degree of sophistication. Although Diophantus and the Babylonians used mostly special ad hoc methods to solve equations, Al-Khwarizmi's contribution was fundamental. He solved linear and quadratic equations without algebraic symbolism, negative numbers or zero, thus he had to distinguish several types of equations.[17]

In the context where algebra is identified with the theory of equations, the Greek mathematician Diophantus has traditionally been known as the "father of algebra" and in the context where it is identified with rules for manipulating and solving equations, Persian mathematician al-Khwarizmi is regarded as "the father of algebra".[18][19][20][21][22][23][24] It is open to debate whether Diophantus or al-Khwarizmi is more entitled to be known, in the general sense, as "the father of algebra". Those who support Diophantus point to the fact that the

algebra found in Al-Jabr is slightly more elementary than the algebra found in Arithmetica and that Arithmetica is syncopated while Al-Jabr is fully rhetorical.[25] Those who support Al-Khwarizmi point to the fact that he introduced the methods of "reduction" and "balancing" (the transposition of subtracted terms to the other side of an equation, that is, the cancellation of like terms on opposite sides of the equation) which the term al-jabr originally referred to,[26] and that he gave an exhaustive explanation of solving quadratic equations,[27] supported by geometric proofs while treating algebra as an independent discipline in its own right.[22] His algebra was also no longer concerned "with a series of problems to be resolved, but an exposition which starts with primitive terms in which the combinations must give all possible prototypes for equations, which henceforward explicitly constitute the true object of study". He also studied an equation for its own sake and "in a generic manner, insofar as it does not simply emerge in the course of solving a problem, but is specifically called on to define an infinite class of problems".[28]

Another Persian mathematician Omar Khayyam is credited with identifying the foundations of algebraic geometry and found the general geometric solution of the cubic equation. His book Treatise on Demonstrations of Problems of Algebra (1070), which laid down the principles of algebra, is part of the body of Persian mathematics that was eventually transmitted to Europe.[29] Yet another Persian mathematician, Sharaf al-D n al-T s , found algebraic and numerical solutions to various cases of cubic equations.[30] He also developed the concept of a function.[31] The Indian mathematicians Mahavira and Bhaskara II, the Persian mathematician Al-Karaji,[32] and the Chinese mathematician Zhu Shijie, solved various cases of cubic, quartic, quintic and higher-order polynomial equations using numerical methods. In the 13th century, the solution of a cubic equation by Fibonacci is representative of the beginning of a revival in European algebra. Ab  al- ,asan ibn  l- l-  al-Qala  d  (1412 1486) took "the first steps toward the introduction of algebraic symbolism". He also computed  $\sqrt[n]{2}$ ,  $\sqrt[n]{3}$  and used the method of successive approximation to determine square roots.[33]

#### Modern history

Fran ois Vi te's work on new algebra at the close of the 16th century was an important step towards modern algebra. In 1637, Ren  Descartes published La G om trie, inventing analytic geometry and introducing modern algebraic notation. Another key event in the further development of algebra was the general algebraic solution of the cubic and quartic equations, developed in the mid-16th century. The idea of a determinant was developed by Japanese mathematician Seki K wa in the 17th century, followed independently by Gottfried Leibniz ten years later, for the purpose of solving systems of simultaneous linear equations using matrices. Gabriel Cramer also did some work on

matrices and determinants in the 18th century. Permutations were studied by Joseph-Louis Lagrange in his 1770 paper "Réflexions sur la résolution algébrique des équations" devoted to solutions of algebraic equations, in which he introduced Lagrange resolvents. Paolo Ruffini was the first person to develop the theory of permutation groups, and like his predecessors, also in the context of solving algebraic equations.

Abstract algebra was developed in the 19th century, deriving from the interest in solving equations, initially focusing on what is now called Galois theory, and on constructibility issues.[34] George Peacock was the founder of axiomatic thinking in arithmetic and algebra. Augustus De Morgan discovered relation algebra in his Syllabus of a Proposed System of Logic. Josiah Willard Gibbs developed an algebra of vectors in three-dimensional space, and Arthur Cayley developed an algebra of matrices (this is a noncommutative algebra).[35]

Areas of mathematics with the word algebra in their name

Some subareas of algebra have the word algebra in their name; linear algebra is one example. Others do not: group theory, ring theory, and field theory are examples. In this section, we list some areas of mathematics with the word "algebra" in the name.

Many mathematical structures are called algebras:

Elementary algebra

1 ∈ power (exponent)

2 ∈ coefficient

3 ∈ term

4 ∈ operator

5 ∈ constant term

$x$   $y$   $c$   $\hat{=}$  variables/constants Algebraic expression notation: 1  $\hat{=}$  power (exponent) 2  $\hat{=}$  coefficient 3  $\hat{=}$  term 4  $\hat{=}$  operator 5  $\hat{=}$  constant term  $\hat{=}$  variables/constants

Elementary algebra is the most basic form of algebra. It is taught to students who are presumed to have no knowledge of mathematics beyond the basic principles of arithmetic. In arithmetic, only numbers and their arithmetical operations (such as  $+$ ,  $\hat{=}$ ,  $\hat{-}$ ,  $\hat{\cdot}$ ) occur. In algebra, numbers are often represented by symbols called variables (such as  $a$ ,  $n$ ,  $x$ ,  $y$  or  $z$ ). This is useful because:

It allows the general formulation of arithmetical laws (such as  $a + b = b + a$  for all  $a$  and  $b$ ), and thus is the first step to a systematic exploration of the properties of the real number system.

$a + b = b + a$  for all  $a$  and  $b$ ), and thus is the first step to a systematic exploration of the properties of the real number system. It allows the reference to "unknown" numbers, the formulation of equations and the study of how to solve these. (For instance, "Find a number  $x$  such that  $3x + 1 = 10$ " or going a bit further "Find a number  $x$  such that  $ax + b = c$ ". This step leads to the conclusion that it is not the nature of the specific numbers that allow us to solve it, but that of the operations involved.)

such that  $3x + 1 = 10$ " or going a bit further "Find a number such that  $a + b = c$ ". This step leads to the conclusion that it is not the nature of the specific numbers that allow us to solve it, but that of the operations involved.) It allows the formulation of functional relationships. (For instance, "If you sell  $x$  tickets, then your profit will be  $3x - 10$  dollars, or  $f(x) = 3x - 10$ , where  $f$  is the function, and  $x$  is the number to which the function is applied".)

Polynomials

The graph of a polynomial function of degree 3

A polynomial is an expression that is the sum of a finite number of non-zero terms, each term consisting of the product of a constant and a finite number of variables raised to whole number powers. For example,  $x^2 + 2x - 3$  is a polynomial in the single variable  $x$ . A polynomial expression is an expression that may be rewritten as a polynomial, by using commutativity, associativity and distributivity of addition and multiplication. For example,  $(x + 1)(x + 3)$  is a polynomial expression, that, properly speaking, is not a polynomial. A polynomial function is a function that is

defined by a polynomial, or, equivalently, by a polynomial expression. The two preceding examples define the same polynomial function.

Two important and related problems in algebra are the factorization of polynomials, that is, expressing a given polynomial as a product of other polynomials that cannot be factored any further, and the computation of polynomial greatest common divisors. The example polynomial above can be factored as  $(x - 1)(x + 3)$ . A related class of problems is finding algebraic expressions for the roots of a polynomial in a single variable.

#### Education

It has been suggested that elementary algebra should be taught to students as young as eleven years old,[36] though in recent years it is more common for public lessons to begin at the eighth grade level (13 y.o.  $\pm$ ) in the United States.[37] However, in some US schools, algebra instruction starts in ninth grade.

#### Abstract algebra

Abstract algebra extends the familiar concepts found in elementary algebra and arithmetic of numbers to more general concepts. Here are the listed fundamental concepts in abstract algebra.

**Sets:** Rather than just considering the different types of numbers, abstract algebra deals with the more general concept of sets: collections of objects called elements. All collections of the familiar types of numbers are sets. Other examples of sets include the set of all two-by-two matrices, the set of all second-degree polynomials ( $ax^2 + bx + c$ ), the set of all two dimensional vectors of a plane, and the various finite groups such as the cyclic groups, which are the groups of integers modulo  $n$ . Set theory is a branch of logic and not technically a branch of algebra.

**Binary operations:** The notion of addition (+) is generalized to the notion of binary operation (denoted here by  $\hat{-}$ ). The notion of binary operation is meaningless without the set on which the operation is defined. For two elements  $a$  and  $b$  in a set  $S$ ,  $a \hat{-} b$  is another element in the set; this condition is called closure. Addition (+), subtraction ( $\hat{'}$ ), multiplication ( $\hat{-}$ ), and division ( $\hat{\cdot}$ ) can be binary operations when defined on different sets, as are addition and multiplication of matrices, vectors, and polynomials.



Identity elements: The numbers zero and one are generalized to give the notion of an identity element for an operation. Zero is the identity element for addition and one is the identity element for multiplication. For a general binary operator  $\hat{-}$  the identity element  $e$  must satisfy  $a \hat{-} e = a$  and  $e \hat{-} a = a$ , and is necessarily unique, if it exists. This holds for addition as  $a + 0 = a$  and  $0 + a = a$  and multiplication  $a \tilde{-} 1 = a$  and  $1 \tilde{-} a = a$ . Not all sets and operator combinations have an identity element; for example, the set of positive natural numbers  $(1, 2, 3, \dots)$  has no identity element for addition.

Inverse elements: The negative numbers give rise to the concept of inverse elements. For addition, the inverse of  $a$  is written  $\hat{'}a$ , and for multiplication the inverse is written  $a\hat{'}1$ . A general two-sided inverse element  $a\hat{'}1$  satisfies the property that  $a \hat{-} a\hat{'}1 = e$  and  $a\hat{'}1 \hat{-} a = e$ , where  $e$  is the identity element.

Associativity: Addition of integers has a property called associativity. That is, the grouping of the numbers to be added does not affect the sum. For example:  $(2 + 3) + 4 = 2 + (3 + 4)$ . In general, this becomes  $(a \hat{-} b) \hat{-} c = a \hat{-} (b \hat{-} c)$ . This property is shared by most binary operations, but not subtraction or division or octonion multiplication.

Commutativity: Addition and multiplication of real numbers are both commutative. That is, the order of the numbers does not affect the result. For example:  $2 + 3 = 3 + 2$ . In general, this becomes  $a \hat{-} b = b \hat{-} a$ . This property does not hold for all binary operations. For example, matrix multiplication and quaternion multiplication are both non-commutative.

## Groups

Combining the above concepts gives one of the most important structures in mathematics: a group. A group is a combination of a set  $S$  and a single binary operation  $\hat{-}$ , defined in any way you choose, but with the following properties:

An identity element  $e$  exists, such that for every member  $a$  of  $S$ ,  $e \hat{-} a$  and  $a \hat{-} e$  are both identical to  $a$ .

Every element has an inverse: for every member  $a$  of  $S$ , there exists a member  $a\hat{'}1$  such that  $a \hat{-} a\hat{'}1$  and  $a\hat{'}1 \hat{-} a$  are both identical to the

identity element.

of  $S$ , there exists a member  $e$  such that  $ae$  and  $ea$  are both identical to the identity element. The operation is associative: if  $a$ ,  $b$  and  $c$  are members of  $S$ , then  $(a \hat{-} b) \hat{-} c$  is identical to  $a \hat{-} (b \hat{-} c)$ .

If a group is also commutative  $\hat{-}$  that is, for any two members  $a$  and  $b$  of  $S$ ,  $a \hat{-} b$  is identical to  $b \hat{-} a$   $\hat{-}$  then the group is said to be abelian.

For example, the set of integers under the operation of addition is a group. In this group, the identity element is 0 and the inverse of any element  $a$  is its negation,  $-a$ . The associativity requirement is met, because for any integers  $a$ ,  $b$  and  $c$ ,  $(a + b) + c = a + (b + c)$

The non-zero rational numbers form a group under multiplication. Here, the identity element is 1, since  $1 \tilde{-} a = a \tilde{-} 1 = a$  for any rational number  $a$ . The inverse of  $a$  is  $1/a$ , since  $a \tilde{-} 1/a = 1$ .

The integers under the multiplication operation, however, do not form a group. This is because, in general, the multiplicative inverse of an integer is not an integer. For example, 4 is an integer, but its multiplicative inverse is  $1/4$ , which is not an integer.

The theory of groups is studied in group theory. A major result of this theory is the classification of finite simple groups, mostly published between about 1955 and 1983, which separates the finite simple groups into roughly 30 basic types.

Semi-groups, quasi-groups, and monoids are algebraic structures similar to groups, but with less constraints on the operation. They comprise a set and a closed binary operation but do not necessarily satisfy the other conditions. A semi-group has an associative binary operation but might not have an identity element. A monoid is a semi-group which does have an identity but might not have an inverse for every element. A quasi-group satisfies a requirement that any element can be turned into any other by either a unique left-multiplication or right-multiplication; however, the binary operation might not be associative.

All groups are monoids, and all monoids are semi-groups.

## Rings and fields

Groups just have one binary operation. To fully explain the behaviour of the different types of numbers, structures with two operators need to be studied. The most important of these are rings and fields.

A ring has two binary operations  $(+)$  and  $(\tilde{-})$ , with  $\tilde{-}$  distributive over  $+$ . Under the first operator  $(+)$  it forms an abelian group. Under the second operator  $(\tilde{-})$  it is associative, but it does not need to have an identity, or inverse, so division is not required. The additive  $(+)$  identity element is written as  $0$  and the additive inverse of  $a$  is written as  $\hat{a}$ .

Distributivity generalises the distributive law for numbers. For the integers  $(a + b) \tilde{-} c = a \tilde{-} c + b \tilde{-} c$  and  $c \tilde{-} (a + b) = c \tilde{-} a + c \tilde{-} b$ , and  $\tilde{-}$  is said to be distributive over  $+$ .

The integers are an example of a ring. The integers have additional properties which make it an integral domain.

A field is a ring with the additional property that all the elements excluding  $0$  form an abelian group under  $\tilde{-}$ . The multiplicative  $(\tilde{-})$  identity is written as  $1$  and the multiplicative inverse of  $a$  is written as  $a\hat{1}$ .

The rational numbers, the real numbers and the complex numbers are all examples of fields.

See also

References

Citations

Works cited

## Reference

[Study Guide for Essentials of Nursing Research: Appraising Evidence for Nursing Practice](#)

[Demystifying Research for Pre-Medical Students: A Comprehensive Guide to Succeeding in Your Research Journey](#)